LECTURES ON SECOND ORDER ELLIPTIC AND PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

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1 INTRODUCTION

The aim of these lectures is to give an introduction to the theory of linear second order elliptic and parabolic partial differential equations. A *partial differential equation of order k* is an equation involving an unknown function u of two or more variables and its derivatives up to order k:

$$F(x, u, Du, \cdots, D^k u) = 0\,. \tag{1.1}$$

Here x denotes the independent variables which typically vary over some domain in a Euclidean space $\mathbb{R}^n$ with $n \geq 2$. Equation (1.1) is said to be *linear* if the left hand side of (1.1) is an affine function of u and its derivatives. Thus a general linear second order partial differential equation can be written in the form

$$Lu = \sum_{i,j=1}^{n} a^{ij}(x) D_{ij} u + \sum_{i=1}^{n} b^i(x) D_i u + c(x) u = f(x)\,. \tag{1.2}$$

Here are some important examples of second order linear equations.

Laplace's equation

$$\Delta u = \sum_{i=1}^{n} D_{ii} u = 0\,. \tag{1.3}$$

Poisson's equation

$$\Delta u = f(x)\,. \tag{1.4}$$

Heat equation

$$\frac{\partial u}{\partial t} = \Delta u\,. \tag{1.5}$$

Wave equation

$$\frac{\partial^2 u}{\partial t^2} = \Delta u\,. \tag{1.6}$$