MULTIWELL PROBLEMS AND RESTRICTIONS ON MICROSTRUCTURE

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§1. Problems and Background.

Consider a variational problem:

$$\min_{u=u_0 on \partial\Omega} \int_\Omega F(Du(x))dx, \tag{1}$$

and suppose the integrand F is continuous and bounded below with reasonable growth at infinity. (1) is wealky lower semicontinuous in $W^{1,p}$ if F is *quasiconvex* (see e.g. Acerbi-Fusco [AF]). By definition this means

$$\int_G F(P + D\phi(x))dx \geq F(P)\,\mathrm{meas}(G),$$

for some domain G and for all $P \in M^{n\times n}$, $\phi \in W_0^{1,\infty}(G; R^N)$.

Integrands F that arise in the study of martenstic phase transitions are not quasiconvex. (1) is not lower semicontinuous and it cannot be solved by the direct method of the calculus of variations. A minimizing sequence can develop spatial oscillations in its gradients Du_k, leading to weak rather than strong convergence. The central idea of the energy minimizing is that these oscillations model the microstructure observed in real materials (Ball-James [BJ1, BJ2]).

The most interested integrands at the moment are those F which have 'multiple well structure', i.e. $F \geq 0$ everywhere and $F = 0$ on a known set K. The connected components of K are 'elastic energy wells'. They represent preferred gradients.

In this talk I will restrict to the situation when K is a finite subset of $M^{n\times N}$, in particular, a finite set of $M^{2\times 2}$ or $M^{3\times 3}$.

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