Degenerate Monge-Ampère equation in algebraic geometry

Hajime Tsuji

1 Introduction

Recently blow up of nonlinear PDE has been studied by many mathematicians in various context. In the blow up of the solution, basically we would like to know :

1. location of the blow up set,
2. description of the singularity.

But in many cases, it is hard to understand the blow up well.

In this paper I would like to study the blow up of parabolic complex Monge-Ampère equations which appear in algebraic geometry. In this case we can describe the blow up very well by algebro-geometric invariants.

A special feature of this equation is the fact that the blow up always occurs along analytic subsets. The singularity is described by Hörmander's L^2 theory for $\bar{\partial}$ operator. I hope that there exists a similar theory for other nonlinear equations.

The paper consists of 5 sections. Section 2 and 3 are devoted to the explanation of Analytic Zariski decomposition (AZD) which we would like to construct in terms of a parabolic complex Monge-Ampère equation. In Section 4 we study the equation and the singularity of the solution. In Section 5, we apply AZD to some problems in algebraic geometry.