Transference from Lipschitz graphs to periodic Lipschitz graphs

by

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§1. Introduction

In this note we will study Fourier multiplier operators on Lipschitz surfaces. On Lipschitz curves the notion of a Fourier transform is initially introduced by R. Coifman and Y. Meyer ([CM]). The monogenic extensions of the exponential functions (see [LMcQ]) enable us to define this notion on surfaces. The paper extends a proof in [GQW] using the monogenic extensions of the Gauss-Weierstrass kernels, and hence proves that the boundedness of certain operators on infinite surfaces can be transferred to the induced operators on periodic surfaces. More general Fourier multipliers rather than the H^∞ ones are considered. For the latter the reader is referred to [McQ1]-[McQ3], [LMcS], [LMcQ] and [GQW].

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§2. Transference from γ to Γ

Denote the standard basis vectors of $\mathbf{R}^{n+1}$ by $e_0, e_1, ..., e_n$, where $e_0^2 = 1, e_i^2 = -1, i = 1, ..., n$, and $e_ie_j = -e_je_i, 1 \leq i < j \leq n$. We then imbed $\mathbf{R}^{n+1}$ into the real Clifford algebra $\mathbf{R}^{(n+1)}$ generated by $e_0, e_1, ..., e_n$, according to which we write a typical $x \in \mathbf{R}^{n+1}$ as $x = \mathrm{x} + x_0e_0$, where $\mathrm{x} = x_1e_1 + \cdots + x_ne_n \in \mathbf{R}^n$. In the sequel we will identify $e_0 = 1$.

We will use the following sets: For $\mu \in (0, \frac{\pi}{2}], \mathbf{C}_{\mu,+} = \{0 \neq x = \mathrm{x} + x_Le_L \in \mathbf{R}^{n+1} \mid x_L > -|\mathrm{x}| \tan \mu\}, \mathbf{C}_{\mu,-} = -\mathbf{C}_{\mu,+}$, and $\mathbf{S}_\mu = \mathbf{C}_{\mu,+} \cap \mathbf{C}_{\mu,-}$.

Let γ be an infinite Lipschitz graph parameterized by

$$\gamma = \{\mathrm{x} + g(\mathrm{x})e_0 | \mathrm{x} \in \mathbf{R}^n, g : \mathbf{R}^n \to \mathbf{R},\ g, \nabla g \in L^\infty(\mathbf{R}^n)\}.$$

Denote by $N = \|\nabla g\|_\infty < \infty$ its Lipschitz constant. Without loss of generality, we assume $-M = \min\{g(\mathrm{x}) | \mathrm{x} \in \mathbf{R}^n\} = -\max\{g(\mathrm{x}) | \mathrm{x} \in \mathbf{R}^n\}, 0 < M < \infty$. Denote $D_l = \sum_{i=0}^n e_i \frac{\partial}{\partial x_i}$ and $D_r = \sum_{i=0}^n \frac{\partial}{\partial x_i} e_i$. For a Clifford-valued function f we define

$$D_l f = \sum_{i=0}^n e_i \frac{\partial f}{\partial x_i}, \quad D_r f = \sum_{i=0}^n \frac{\partial f}{\partial x_i} e_i.$$