The maximum principle for degenerate parabolic PDEs with singularities

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1. Introduction

This is a preliminary version of [7]. Here we shall be concerned with the degenerate parabolic partial differential equation (PDE in short)

(1) $$u_t + F(Du, D^2u) = 0 \quad \text{in } \Omega \times (0,T).$$

Here and in what follows Ω is a domain of $\mathbf{R}^N$, $T > 0$ is a given constant, u represents the real unknown function on $\Omega \times (0,T)$ and F is a real function on $\mathbf{R}^N \times \mathbf{S}^N$, where $\mathbf{S}^N$ denotes the set of real $N \times N$ symmetric matrices.

Recent developments have revealed that equations (1) with F having singularities or discontinuities are important in the study of generalized evolutions of hypersurfaces, especially, in the level set approach.

Chen, Giga and Goto [3] and Evans and Spruck [5] initiated the level set approach, on a firm mathematical basis, to evolutions of hypersurfaces driven by their mean curvature or by some other geometric quantitives alike. In the case of evolution by mean curvature, F turns out to be

$$F(p,X) = -\mathrm{tr}\left(I - \frac{p \otimes p}{|p|^2}\right) X.$$

Thus in their approach $F(p,X)$ is not defined for $p = 0$.

According to Angenent and Gurtin [1, 2], equations (1) with F discontinuous in a set of directions of p's arise in a mathematical model for the dynamics of a melting solid, where the boundary between the solid and liquid phases gives an evolving hypersurface.