Multivalued spatial discretization of dynamical systems *

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Introduction

Suppose that a mapping $f: \ \Omega \mapsto \Omega$, on a compact metric space Ω, generates a discrete dynamical system $\{f^n(x) : \ n = 0, 1, 2, \dots\}$ with chaotic behavior. For brevity we will refer to the system f. Standard computer models of this system are dynamical systems ψ defined on a finite subset $\mathbf{L}$ of Ω. The set of all trajectories of an individual system ψ can differ dramatically from that of the original system even for a fine discretization. For instance, the mapping $f(x) = 2x \,(\bmod 1)$ is chaotic with a unique absolutely continuous invariant measure and cycles of all periods. Yet every N–digital binary discretization ψ_N, defined as the restriction of f to the set $\{i/2^N : i = 1, \dots, 2^{N-1}\}$ is asymptotically trivial, $\psi_N^k \equiv 0$, $k \geq N$. Such effects are an inevitable consequence of discretization in the sense that there always exists some discretization which collapses a given system f onto a given f–invariant set, in particular onto a fixed point or cycle [6].

Degenerate, collapsing behavior such as this can be eliminated by instead modelling with systems φ which can be regarded as either stochastic or multivalued perturbations of the original system f. However, the choice of an appropriate model system φ introduces a conundrum which frequently arises in the theory of ill–posed problems. If the perturbation that is introduced is too large, then the behaviour of the system φ, while not degenerate can differ markedly from f. On the other hand, if the perturbation is not strong enough, collapsing effects will not be avoided. Consequently, questions about the robustness of systems to various levels of stochastic or multivalued perturbation are very important.

Fundamental theoretical results concerning stochastic perturbations can be found in [11, 1, 2]. Approaches using multivalued systems are much less investigated. However, this approach seems to be efficient in investigating systems with fast changing and discontinuous characteristics. Often the most adequate mathematical descriptions of

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