TWO NOTES ON SUPERCRITICAL PROBLEMS ON LONG DOMAINS

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In this short paper, we discuss two results for the problem

$$\begin{aligned} -\Delta u &= \lambda f(u) \quad \text{on } D_n = \widetilde{\Omega} \times (-n, n) \\ u &= 0 \qquad\quad \text{on } \partial D_n \end{aligned} \tag{1}$$

for large n. Here $\widetilde{\Omega}$ is a bounded domain in R^{m-1} with smooth boundary. In [2], we showed that the unstable solutions (unstable for the natural corresponding parabolic) are largely determined by a problem on the infinite string $\widetilde{\Omega} \times R$. Thus there seems no reason that they should behave like the solutions of the corresponding problem on $\widetilde{\Omega}$. (As seen in [2], this contrasts with the stable solutions.) Here we present an example for each m with $m \geq 3$ where the unstable solutions of (1) for all n are quite different to those of the lower dimensional problem and this holds uniformly in n. To do this, we consider f's which are asymptotically like y^q for large y where the nonlinearity is subcritical on $\widetilde{\Omega}$ but supercritical on D_n. Note that for many f's general results in nonlinear functional analysis ensure a general resemblance between the solution structure on $\widetilde{\Omega}$ and on D_n. Secondly, we show that on certain symmetric domains, we can bound the branch of symmetric solutions uniformly in n where the nonlinearity grows faster than the critical nonlinearity.

Here we choose q such that $(m+2)/(m-2) < q < (m+1)/(m-3)$ and consider f smooth convex such that $f(0) > 0$, $f(y) > 0$ for $y > 0$ and $f(y) \sim y^q$ as $y \to \infty$.