FINITE ELEMENT METHODS FOR IDENTIFICATION OF PARAMETERS IN PARABOLIC PROBLEMS

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1.INTRODUCTION

In the present paper, we examine finite element Galerkin methods for the identification of unknown parameters in parabolic partial differential equations, and derive a number of optimal error estimates. In the first part of the paper, we consider the problem of finding, for a homogeneous material, a control parameter $p(t)$, along with the temperature distribution $U(x,t)$, which yields a specified energy trajectory $E(t)$ (see (1.4) below) pescribed on the whole of its spatial domain. The corresponding model gives rise to the following inverse problem: find $p(t)$ and $U(x,t)$ such that

$$U_t - U_{xx} + p(t)U_x = \tilde{f}(x,t), \quad (x,t) \in (0,1) \times (0,T], \tag{1.1}$$

$$U(x,0) = U_0(x), \quad x \in [0,1], \tag{1.2}$$

$$U(0,t) = \tilde{f}_1(t), \qquad U(1,t) = \tilde{f}_2(t), \quad t \in [0,T] \tag{1.3}$$

along with the over–specificed total internal energy condition

$$\int_0^1 U(x,t)\,dx = E(t), \quad t \in [0,T], \tag{1.4}$$

where $\tilde{f}, \tilde{f}_1, \tilde{f}_2, U_0$ and E are given functions of their arguments with $\tilde{f}_2 - \tilde{f}_1 \neq 0$.

In the literature, output error criteria procedures are regularly proposed and analysed (Banks and Kunish [**2**]) for the solution of inverse problems, including ones like that which is formulated above. The basis for such indirect approaches is optimization: the closeness (defined by a given objective function) with which computed estimates can be matched with observations of measurable quantities is minimized with respect to some admissable class of control parameters. Least squares is often used to define the closeness (objective function). The resulting methods are referred to as least squares output error criteria procedures.