MULTIGRID METHODS FOR TOMOGRAPHIC RECONSTRUCTION

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1. INTRODUCTION

In many inversion problems, the random nature of the observed data has a very significant impact on the accuracy of the reconstruction. In these situations, reconstruction techniques that are based on the known statistical properties of the data, are particularly useful. In a report [1], we considered methods for the reconstruction of an object from its projection data, and showed that maximum likelihood methods are particularly successful when the data is very noisy. Although maximum likelihood methods were first applied to emission tomography [2], they have also found utility in other imaging modalities, including transmission [3], diffraction [4] and limited angle transmission tomography [5, 6]. The major problem with maximum likelihood methods for image reconstruction, is that the normal technique used, the expectation-maximisation (EM) algorithm, is excruciatingly slow to converge to the desired objective. In this paper, we demonstrate the use of multigrid methods to overcome this handicap.

In order to facilitate subsequent discussion, we shall give a brief review of the expectation-maximisation (EM) algorithm, as described in [1]. The EM algorithm is an algebraic reconstruction technique where we resolve the object to be reconstructed into a grid-like array of unknowns and set up algebraic equations for these unknowns in terms of the measured projection data. The reconstruction region is sub-divided into pixels numbered 1, ..., B. In pixel j, the parameter to be reconstructed is assumed to have a constant value λ_j. If the projections are measured at sites numbered 1, ..., T, then this linear system of equations can then be written as :

$$\mathbf{n} = \mathbf{A}\boldsymbol{\lambda} + \boldsymbol{\varepsilon} \tag{1.1}$$

where **A** is a sparse matrix connecting the object vector, $\boldsymbol{\lambda} \in \mathbb{R}^B$, to the measured data vector, $\mathbf{n} \in \mathbb{R}^T$, and $\boldsymbol{\varepsilon} \in \mathbb{R}^T$, is the vector of errors due to inaccuracies in both the measurement and the discretization processes.

The reconstruction problem is to find a method of estimating the object vector, $\boldsymbol{\lambda}$, given only the measurements, **n**, and some estimate of the connection matrix, **A**. In transmission tomography, the matrix element A_{ij} linking pixel j to detector i is calculated as a function of the intersection area of the ray path through pixel j to the relevant detector. In emission tomography, A_{ij} is usually calculated as some approximation to the probability that detector i measures activity in pixel j. The algorithms to solve (1.1) can require a large amount of computation as a result of the characteristics of the matrix **A** :

- it is huge, possibly as large as $10^5 \times 10^5$ or more,
- it is reasonably sparse, perhaps only 10% of the elements are non-zero,
- the pattern or structure of the non-zero elements in the matrix is irregular and therefore difficult to exploit,
- the location and value of the non-zero elements may be efficiently generated either by rows or by columns, depending on the definition of A, but it is generally not possible to do so for both rows and columns.