METHODS FOR CHOOSING THE REGULARIZATION PARAMETER

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1. INTRODUCTION

Many inverse problems arising in practice can be modelled in the form of an operator equation

$$Kf = g, \tag{1.1}$$

where the function $g : \mathbb{R}^d \to \mathbb{R}$ is known only as discrete noisy data $y_i = g(x_i) + \epsilon_i$, $i = 1, \ldots, n$. An example is the estimation of an unknown parameter function f (e.g. a diffusivity) in a partial differential equation $L_f u = h$. Here the operator equation (1.1) is $Kf = u(f) = g$, where $u(f)$ is the solution of the forward problem $L_f u = h$ subject to some boundary conditions, but g is known only at discrete points and with error.

Suppose that equation (1.1) has a unique solution f_0. In many cases (1.1) is ill–posed, meaning that with respect to appropriate normed spaces, the inverse of K is not continuous at f_0. To obtain a reasonably good approximate solution, it is usually necessary to stabilize the problem. This is especially true given only discrete noisy data.

A well-known and effective technique for stabilizing the problem is the method of regularization. This replaces the original problem by the minimization over f in a