Appendix C

Maximal developments.

In this Appendix we shall discuss the existence of maximal developments, and we shall also prove some criteria which allow one to decide whether or not a given development is maximal.

C.1 Existence of maximal space–times.

In this section we shall prove the existence of maximal space–times; the reader should note that we are not making any global hyperbolicity hypotheses. The arguments here follow essentially those of [21]. Throughout this section "W manifold" stands for a connected, paracompact, Hausdorff n–dimensional manifold of differentiability class W such that $W \subset C^1$, where W stands for *e.g.* $C^{k,\alpha}$ or some Sobolev class, etc. The manifold will be said Lorentzian if it is equipped with a metric tensor, perhaps defined only almost everywhere, of a differentiability class adapted to that of W. For example, if $W = C^{k,\alpha}$ then we should have $k \geq 1$ and the metric, defined everywhere, will be of $C^{k-1,\alpha}$ differentiability class. It is useful to keep in mind that W can be a rather complicated space, *e.g.* for the purpose of the Cauchy problem in general relativity an appropriate space W is the set of maps which preserve the condition that the components of the metric tensor $\gamma_{\mu\nu}$ restricted to the hypersurfaces $\Sigma = \{t = \text{const}\}$ are of Sobolev class $H^k_{loc}(\Sigma)$, the time–derivatives of $\gamma_{\mu\nu}$ are in $H^{k-1}_{loc}(\Sigma)$, etc. A W Lorentzian manifold