Appendix B

On a class of $U(1) \times U(1)$ symmetric metrics found by V.Moncrief.

In this Appendix we shall prove that "strong cosmic censorship" holds in a six parameter family of *non–polarized* $U(1) \times U(1)$ symmetric metrics found by Moncrief[1] [93]. Apart from being interesting in their own right, these metrics provide a good testing ground for various *a priori* estimates one can obtain for general $U(1) \times U(1)$ symmetric metrics, *cf.* Chapter 3.

Throughout this Appendix the letter C denotes a constant the value of which may vary from line to line.

B.1   A harmonic map problem.

Let $x(t, \theta) = (\rho(t, \theta), \phi(t, \theta))$ be a map from two–dimensional Minkowski space to a two dimensional constant mean curvature hyperoboloid, set

$$X_t = \frac{\partial x^A}{\partial t} \frac{\partial}{\partial x^A} = \frac{\partial \rho}{\partial t} \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial t} \frac{\partial}{\partial \phi}, \quad X_\theta = \frac{\partial x^A}{\partial \theta} \frac{\partial}{\partial x^A} = \frac{\partial \rho}{\partial \theta} \frac{\partial}{\partial \rho} + \frac{\partial \phi}{\partial \theta} \frac{\partial}{\partial \phi} \ .$$

On the hyperboloid one can introduce coordinates in which the metric takes the form

$$ds^2 = d\rho^2 + \sinh^2 \rho \, d\phi^2 \ .$$

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[1]A similar class of harmonic maps has been considered independently by Shatah and Tahvildar-Zadeh in [118]; *cf.* also [63].