Appendix A

On the "hyperboloidal initial data", and Penrose conditions.

Let us briefly recall the conformal framework introduced by Penrose [104] to describe the behaviour of physical fields at null infinity. Given a, say vacuum, smooth "physical" space-time $(\tilde{M},\tilde{\gamma})$ one associates to it a smooth "unphysical space-time" (M,γ) and a smooth function Ω on M, such that $\tilde{M}$ is a subset of M and

$$\Omega|_{\tilde{M}} > 0\,, \quad \gamma_{\mu\nu}|_{\tilde{M}} = \Omega^2\tilde{\gamma}_{\mu\nu}\,, \tag{A.0.1}$$

$$\Omega|_{\partial\tilde{M}} = 0\,, \tag{A.0.2}$$

$$d\Omega(p) \neq 0 \quad \text{for} \quad p \in \partial\tilde{M}\,, \tag{A.0.3}$$

where $\partial\tilde{M}$ is the boundary of $\tilde{M}$ in M (it should be stressed that in this section a notation *inverse* to that used in 1.6 is used: tilded quantities denote the physical ones, while non–tilded quantities denote the unphysical (conformally rescaled) ones). It is common usage in general relativity to use the symbol $\mathcal{I}$ for $\partial\tilde{M}$, and we shall sometimes do so. If Σ is a hypersurface in M, by $\mathcal{I}^+$ we shall denote the connected component of $\mathcal{I}$ which intersects the causal future of Σ. The hypothesis of smoothness of (M,γ,Ω) and the fact that $(\tilde{M},\tilde{\gamma})$ is vacuum imposes several restrictions on various fields; if one defines (*cf.* [104])

$$P_{\mu\nu} = \tfrac{1}{2}\left(R_{\mu\nu} - \tfrac{1}{6}R\gamma_{\mu\nu}\right), \tag{A.0.4}$$