Chapter 1

Introduction

In the last ten years or so a considerable amount of work has been done to transform general relativity into a mathematically rigorous discipline. With the work of Christodoulou and Klainerman [29] on stability of Minkowski space–time, the work of Schoen and Yau [113] on the positive energy theorem, the work of Christodoulou [26] [27] on the gravitational collapse, the work of Newman [100] and others (*cf.* *e.g.* [55] and references therein) on Yau's Lorentzian splitting conjecture, the work of Bartnik [7] on maximal hypersurfaces in Lorentzian manifolds, general relativity has become a respectable field of mathematical research. For an analyst interested in differential geometry, general relativity turns out to be a rich source of various, sometimes extremely difficult, mathematical problems, encompassing all classical classes of partial differential equations — hyperbolic (*cf.* *e.g.* [49] [29] [53]), elliptic (*cf.* *e.g.* [19] [7] [9] [74] [127] [3] [2]), and even parabolic (*cf.* *e.g.* [8] [33]), as well as some difficult problems of the theory of dynamical systems (*cf.* *e.g.* [12] [121]). The aim of this paper is to present to a mathematically oriented reader one of the current research problems in general relativity — the problem of uniqueness in the large of solutions of Einstein's equations, also known under the baroque name of "strong cosmic censorship".

In this chapter we discuss some old and new results on global structure of space–time.