The Calculation of Eigenvalues for Nonadiabatic Oscillations in Magnetized Polytropic Atmospheres

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1. Introduction

Linear oscillations in magnetized polytropic atmospheres are investigated by means of a normal mode analysis. For nonadiabatic motions, in the presence of a uniform vertical field , the governing differential equations are sixth order. Subject to appropriate boundary conditions, the system is an eigenvalue problem in which the eigenvalue appears nonlinearly. Discretization of the equations results in a matrix problem for which the eigenvalues may be determined. Previous investigations ([1], [4]) have used root finding techniques on the determinant of the relevant lambda matrix. Here we return to this problem and investigate the eigenspectrum by means of a generalized eigenvalue problem. We were motivated to look at the problem in this way since such an approach requires no initial approximation and no a priori knowledge of the structure of the spectrum. Previously we have found that, in the case adiabatic oscillations in which the eigenvalue appears linearly, a matrix method approach, using the standard QR algorithm, has been successful in determining the complete eigenspectrum ([16], [17]).

Magnetic fields permeate the solar surface layers and extend into the solar atmosphere and there is a large body of observational evidence concerning the oscillations which occur in these magnetized, stratified atmospheres. Solar oscillatory motions have been extensively investigated since photospheric oscillations were first discovered by Leighton,Noyes and Simon [10]. The behaviour of the oscillations differs depending on whether the observations are performed within the quiet photosphere or within sunspots. In the absence of magnetic field the quiet photosphere vibrates acoustically with an oscillation period of approximately five minutes; these are the ubiquitous p-modes. In sunspots the presence of an intense magnetic field modifies the oscillatory behaviour giving rise to essentially three types of motion as identified by their periods. In sunspot umbrae, where the magnetic field is nearly vertical relative to the solar surface, the oscillatory motions have two distinct sets of periods; one se

One approach in the theoretical investigations of these oscillations has been to model the solar atmosphere as a slab of gas of fixed vertical extent but infinite in the horizontal direction. In these plane parallel models the atmosphere is stratified by gravity and when modelling a sunspot the atmosphere is permeated by a constant magnetic field. The normal modes of a particular atmospheric model are evaluated and these modes are compared with the observed oscillations. Within the framework of these models both adiabatic and nonadiabatic motions have been studied ([1], [3], [6], [8], [9], [14], [15], [16] and [18]).

For adiabatic motions in the presence of a vertical magnetic field the governing system of differential equations are of order four. For such a system it is only when the equilibrium atmosphere is isothermal is it possible to obtain analytic solutions. For other