

**NOTES ON THE ISOMETRIC EMBEDDING PROBLEM
AND THE NASH-MOSER IMPLICIT FUNCTION
THEOREM**

BEN ANDREWS

CONTENTS

1. Introduction and summary	158
1.1. General remarks	158
1.2. The isometric embedding problem	160
1.3. Perturbation of embeddings	160
1.4. Freeness of embeddings and immersions	162
1.5. Nash's perturbation result	162
1.6. Loss of differentiability	164
2. Setting up the isometric embedding	164
2.1. Difficulties in applying the perturbation result	164
2.2. Nash's y and z embeddings	165
2.3. Existence of free embeddings	165
3. Approximate isometric embeddings	169
3.1. The Nash Twist	170
3.2. Applying the Nash Twist	171
3.3. Existence of Full maps	172
3.4. Isometric embedding in high dimensions	173
3.5. Nash's argument	174
3.6. C^1 isometric embeddings	174
4. Smoothing operators on manifolds	175
4.1. The required estimates	176
4.2. Mollifications	176
4.3. Reduction to the Euclidean case	176
4.4. Nash's smoothing operators	177
4.5. Smoothing estimates	178
4.6. Approximation estimates	179
4.7. Approximating tensors	181
5. Perturbation result after Hörmander	181
5.1. Decomposition into frequency bands	181
5.2. A characterisation of $C^{k,\alpha}$ functions	182
5.3. The approximation process	184