DILATION OF CONTRACTIVE TUPLES: A SURVEY

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0. INTRODUCTION

In this survey article we start with the unitary dilation of a single contraction due to Sz.-Nagy and Foias [46]. Ando gave a generalization to a pair of commuting contractions [2]. He proved that such a pair has a simultaneous commuting dilation. Then came the startling revealation from Varopoulos [47], Parrott [37] and Crabb-Davie [19] that this phenomenon can not be generalized any further. They produced examples of triples of commuting contractions which fail to have any commuting isometric dilation. The next stage of developments saw the successful attempt of dilating a special class of tuples, viz., the contractive ones. Drury [28], in connection to his generalization of von Neumann's inequality, and then Arveson [6] proved the *standard commuting* dilation for commuting contractive tuples. Several authors pursued the idea of dilating any contractive tuple (commuting or not) to isometries with orthogonal ranges. Some ideas along this direction can already be seen in an early paper of Davis [22]. In more concrete form this dilation appeared in the papers of Bunce [18] and Frazho [29]. A real extensive study of this notion has been carried out by Popescu in a series of papers, see [41] - [45] and also [3], [4] with Arias. He has neat generalizations of many results from one variable situation. This dilation is called the *standard non-commuting* dilation. Davidson, Kribs and Shpigel [21] derive more information about this dilation for finite rank tuples. Then of course arose the natural question that if one starts with a commuting contractive tuple, then what is the relation between the two dilations that it posseses. A recent article by Bhat, Bhattacharyya and Dey [16] show that the standard commuting dilation is the maximal commuting dilation sitting inside the standard non-commuting dilation.

Section 1 is about unitary dilation of a contraction and von Neumann's inequality. Simultaneous commuting unitary dilation of a pair of commuting contractions and the von Neumann inequality for such pairs is taken up in Section 2. In this section, we also show that in general a triple of commuting contractions does not have a commuting unitary dilation. In Section 3, the contractive tuples are introduced