MOVING BOUNDARY PROBLEMS WITH NONLINEAR DIFFUSION

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1. INTRODUCTION

In keeping with the central topic of this mini–conference, I'll be discussing two moving boundary problems that originate in practical nonlinear diffusion models. In each case, the results have direct application to environmental hydrology.

The first moving boundary problem represents the absorption of water by the soil under a pond. In this case, the moving boundary is the free interface between the saturated and unsaturated zones. My exact solution to this problem is yet to appear in the literature [1].

In the second problem, the moving boundary arises indirectly. Here, we consider infiltration of water at constant rate into a porous stratum of finite depth, underlain by an impermeable barrier. The appropriate boundary value problem is highly nonlinear, in both the governing equation and the boundary conditions. For a particular choice of soil hydraulic properties, this nonlinear problem on a rigid domain may be transformed to a linear boundary value problem on a domain that shrinks linearly in time. This is how the moving boundary problem originates. Our exact solution to this problem appeared two years ago [2]. However, our method of solution has not yet been justified rigorously. We are able to throw some light on this problem here. Also, we are now able to solve the model of water infiltration into a finite soil column that is already partially wetted. Previously we assumed zero initial water content and avoided the more difficult situation of gravity–induced redistribution occurring in the early stages of infiltration.