A PRIORI ERROR ESTIMATES FOR FINITE ELEMENT GALERKIN APPROXIMATIONS TO A FREE BOUNDARY PROBLEM IN POLYMER TECHNOLOGY

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1. INTRODUCTION.

In this paper, we examine a finite element Galerkin method for a free boundary problem arising in the polymer industry which models the penetration of a solvent into a glassy polymer. Initially, we briefly discuss the model proposed by Astarita and Sarti[**1**], and the related existence and uniqueness results.

Consider a semi-infinite slab of glassy polymer occupying the half space $x \geq 0$, which is in contact with a solvent. When the solvent concentration at the face of the polymer at $x = 0$ exceeds some threshold, the solvent moves into the polymer creating a swollen zone through which the solvent diffuses according to Fick's law. The interface that is the free boundary between the swollen zone and the glassy polymer obeys an empirical penetration law, which guarantees that the speed of the penetration increases with the excess of concentration above the threshold. In addition, in order to fully specify the free boundary, an additional condition must be imposed in the form of a mass conservation condition. In the present model, we assume that the penetration process has commenced so that a free boundary has already formed and that the swelling takes place instantaneously at the interface. With appropriate non-dimensional normalised variables, the above model leads to the following parabolic free boundary problem.

Problem $\tilde{\mathbf{P}}$. Find a pair $\{U(y,t), s(t)\}$ such that

$$U_t - U_{yy} = 0,\ 0 < y < s(t), \quad 0 < t \leq T \tag{1.1}$$

$$U(y,0) = g(y), \quad 0 < y \leq 1, \tag{1.2}$$