WATER NON-WAVES

E.O. Tuck

This paper is about free-surface problems of the type that usually produce water waves. That is, it is about boundary-value problems for Laplace's equation in a water-occupied domain, with a free surface between water and air, which is influenced by gravity g, and therefore on which a quadratically nonlinear constant-pressure condition holds.

A large class of such problems is such that the free surface is plane when undisturbed: for example, the plane surface of a calm sea. A small disturbance can then produce small waves, which are asymptotically sinusoidal and linear. A large disturbance can produce large waves, which can be periodic but non-sinusoidal – nonlinear Stokes waves.

But waves are not essential, whether linear or nonlinear. For example, consider the effect on a stream U in a two-dimensional flow, of a disturbance created by a small symmetrical over-pressure $P(x)$, as could be caused by blowing air on the water surface over a finite segment $-\ell < x < \ell$, as in Figure 1. This could be a model of a (rather wide!) hovercraft.

This pressure disturbance $P(x)$ certainly deforms the free surface, and in general creates a wave trailing behind it, as $x \to +\infty$. The linearised theory of water waves predicts (Vanden-Broeck and Tuck [27]) that this trailing wave has amplitude proportional to

$$A = \int_{-\ell}^{\ell} P(x) \cos(\kappa x)\, dx$$