PARTICULAR SOLUTIONS METHODS FOR FREE AND MOVING BOUNDARY PROBLEMS

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1 Introduction

Consider a quasilinear partial differential equation

(1) $$Lu = f \qquad \text{on } S$$

subject to a fixed boundary condition

(2) $$B_1 u = g \qquad \text{on } \Gamma$$

where Γ is the boundary of the region S. Suppose that u is approximated in the form

(3) $$u \simeq u_n = \sum_{j=1}^{n} c_j \psi_j(x, y)$$

where ψ_j is a particular solution of Eq. 1 for every j. Suppose that the parameters c_j in Eq. 3 are determined by requiring that u_n should satisfy Eq. 2 approximately according to a chosen interpolation or least squares (or similar) fitting criterion.

In the case of a free boundary problem, the boundary Γ is unknown, and a second boundary condition

(4) $$B_2\, u = h \qquad \text{on } \Gamma$$

is specified. Some form of iterative procedure is then applied to determine a u_n of form Eq. 3 and an approximation Γ_n to Γ such that, in the limit, u_n satisfies both Eq. 3 and Eq. 4.

Such a "method of particular solutions" has been adopted successfully for free boundary problems by [6], and the aim of the present paper is to expose and further expand on this work. In particular a broader variety of particular solutions is provided, and a wider range of test problems is considered. Specifically a family of piecewise polynomial solutions of the heat equation, which coincide with a spline in t on $x = \pm 1$, is introduced, which provides a robust basis for Stefan and related problems. Further, an advection-diffusion (bush-fire) problem is tackled, using separable variable solutions based on modified Bessel functions and a least squares fitting method on the boundary.

Collocation methods are illustrated in a number of effective applications, but, by way of a cautionary tale, an example is given in which an apparently well chosen collocation method leads to divergent approximations while a least squares method converges.