RELAXED VARIATIONAL PRINCIPLES AND ALGORITHMS FOR THE EQUILIBRIA OF ROTATING SELF-GRAVITATING FLUIDS

*Giles Auchmuty**

1. Introduction.

In this paper, we shall analyze some aspects of the variational principle for the axisymmetric equilibria of a rotating self-gravitating fluid. The analysis is developed to justify certain relaxed Lagrangean formulations of the problem and to describe an iterative algorithm for computing the solutions.

These equilibrium solutions provide simple, hydrodynamical models of stars and planets. The physical basis of these theories is described in Tassoul [7]. Here our interest is in developing convergent algorithms for finding these solutions.

In sections 2 to 5 we describe a variational principle for these equilibria, analyze the functional involved, derive the extremality conditions and prove that the local minimizers of this problem are, in fact, classical solutions of the hydrodynamical equations. The existence of such minimizers is proven. Much of this analysis is a variation of that of [1] but there are a number of new results, including convexity theorems, that are important for our purposes.

In section 6 we introduce, and analyze, a relaxed Lagrangean for this problem which converts the variational principle into one of minimizing a functional which is convex in each of two variables separately. This formulation is then used in section 7 to develop an algorithm which generates strict descent sequences for the problem and the convergence of this algorithm is studied.

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