HYPERGROUPS AND HARMONIC ANALYSIS

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0. INTRODUCTION

The modern approach to harmonic analysis on a Lie group treats the representations of the group as the central objects of study, while characters are treated as important but auxiliary objects associated to representations. This is in direct contrast to the modern approach to harmonic analysis on a finite group, which treats the determination and study of the characters of the group as the primary problem, and considers representations as important but auxiliary objects associated to characters.

For finite groups, the reason for this is three fold; 1) the determination of the irreducible characters is a vastly simpler problem than the determination of the irreducible representations 2) almost all of the standard problems of harmonic analysis may be answered solely by means of the character theory and 3) historically, the theory of characters has preceded that of the theory of representations.

This suggests the following interesting question – might it be possible to develop harmonic analysis on a Lie group as essentially a theory of characters, and thereby finesse the present difficulties and technicalities in modern representation theory? [This is not an entirely new idea – in fact Harish Chandra's pivotal work on the existence of discrete series for non-compact semi-simple groups (see for example Varadarajan [8])