ON THE UNIFORM CLASSIFICATION OF $L_p(\mu)$ SPACES

by Anthony Weston *

In this paper we survey results on the uniform classification of $L_p(\mu)$ spaces, we cite several open problems and we tie some loose ends in the existing theory (i.e., Theorem 12(a), (b)).

The topological classification of Banach spaces was initiated in Mazur [17] where he proved

Theorem 1: *For $1 \leq p, q < \infty$ the real Banach spaces $L_p(0,1)$, $L_q(0,1)$ and ℓ_q are homeomorphic.*

From Mazur's work it also followed that the unit balls $B(L_p(0,1))$, $B(L_q(0,1))$ and $B(\ell_q)$ are uniformly homeomorphic. We recall that a bijection $f : X \to Y$ between metric spaces is called a uniform homeomorphism if it is uniformly continuous in both directions.

For a thorough study of the topological structure of linear metric spaces we refer the reader to Bessaga and Pelczynski [8]. We would like to mention that [8] includes a proof of the

Anderson-Kadec Theorem: *Every infinite dimensional, separable, locally convex, complete linear space is homeomorphic to the Hilbert space ℓ_2.*

The papers that led to this theorem are Kadec [13], [14], Anderson [4] and Bessaga and Pelczynski [6], [7]. Note also Torunczyk's generalization in [19]: two Banach spaces are homeomorphic if and only if they have the same density character.

In the present work our interest is in the uniform classification of $L_p(\mu)$ spaces. Combining results of Lindenstrauss [15] and Enflo [10] we get

Theorem 2: *An infinite dimensional $L_{p_1}(\mu_1)$ is not uniformly homeomorphic to $L_{p_2}(\mu_2)$ if $p_1 \neq p_2$, $1 \leq p_i < \infty$.*

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