FRAGMENTABILITY OF ROTUND BANACH SPACES

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Introduction

A topological space X is said to be *fragmented by a metric ρ* if for every $\varepsilon > 0$ and every subset Y of X there exists a nonempty relatively open subset U of Y such that ρ–diam(U) $< \varepsilon$. The space is said to be *fragmentable* if there exists such a metric.

The concept of fragmentability was introduced by Jayne and Rogers in 1985 [1]. Further work has been done by Namioka [2], and Ribarska [3, 4]. A major application of fragmentability is in the study of differentiability of convex functions.

A continuous convex function ϕ on an open convex subset A of a Banach space X is said to be *Gâteaux differentiable* at $x \in A$ if

$$\lim_{\lambda \to 0} \frac{\phi(x+\lambda y) - \phi(x)}{\lambda}$$

exists for all $y \in X$. The function is said to be *Fréchet differentiable* at x if the above limit exists and is approached uniformly for all $y \in X$, $\| y \| = 1$. A Banach space X is said to be *Asplund* (*weak Asplund*) if every continuous convex function on an open convex domain is Fréchet (Gâteaux) differentiable on a dense G_δ subset of its domain.

In 1975 Namioka and Phelps, [5, p.739], established that a Banach space is Asplund if and only if the weak * topology on its dual is fragmented by the dual norm, although their result was not couched in these terms. Weak * fragmentability of the dual by some metric is still the most general known condition implying a Banach space is weak Asplund, [3]. Ribarska, [4], has shown that rotund Banach spaces are weakly fragmentable, and that Banach spaces with rotund dual have weak * fragmentable dual, and hence are weak Asplund. Using techniques developed by Preiss, Phelps and Namioka, [6], she has also shown that a Banach space with an equivalent norm Gâteaux differentiable away from 0 has a weak * fragmentable dual, and hence is weak Asplund. Whether or not weak Asplund spaces are characterized by having a weak * fragmentable dual remains an open question.

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