WEAK COMPACTNESS IN SPACES OF LINEAR OPERATORS

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Let X be a locally convex Hausdorff space (briefly, lcs) and $L(X)$ be the space of all continuous linear operators of X into itself, equipped with the topology of pointwise convergence in X. An element ξ of the dual space $(L(X))'$, of $L(X)$, is the form

$$\xi : T \mapsto \sum_{j=1}^{n} \langle Tx_j, x'_j \rangle, \qquad T \in L(X),$$

for some finite subsets $\{x_j\}_{j=1}^{n} \subseteq X$ and $\{x'_j\}_{j=1}^{n} \subseteq X'$. So, the weak topology of the lcs $L(X)$ is the weak operator topology. Despite this simple description, it is often difficult to determine the relative weak compactness of subsets of $L(X)$. However, to determine the relative weak compactness of subsets of the underlying space X may be easier. So, if $\mathcal{A}$ is a subset of $L(X)$, then a natural starting point would be to examine the relative weak compactness, in X, of the sets $\mathcal{A}[x] = \{Tx; T \in \mathcal{A}\}$, $x \in X$, and relate this to $\mathcal{A}$ as a subset of $L(X)$. Call a family of operators $\mathcal{A} \subseteq L(X)$ pointwise (relatively) weakly compact whenever the subsets $\mathcal{A}[x]$, $x \in X$, of X, are (relatively) weakly compact.

PROPOSITION 1. *Let $\mathcal{A}$ be a subset of $L(X)$.*

(i) If $\mathcal{A}$ is relatively weakly compact, then it is also pointwise relatively weakly compact.

(ii) If $\mathcal{A}$ is equicontinuous, then it is relatively weakly compact if, and only if, it is pointwise relatively weakly compact.

1991 *Mathematics Subject Classification.* 46A32, 46A50, 47B10