

THE HEAT KERNEL FOR H -TYPE GROUPS

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1. INTRODUCTION

This note is a summary of results obtained by the author with the help and guidance of A.H. Dooley. The proofs will appear elsewhere.

These results concern the heat kernel on H -type groups, a class of two step nilpotent simply connected Lie groups which generalize the Heisenberg group. The two-step nilpotent group that appears in the Iwasawa decomposition of a rank one semi-simple Lie group is an H -type group. For more details on H -type groups see Kaplan [6].

Theorem 1 gives an explicit formula for the heat kernel on an H -type group. Folland [2] has shown that for stratified nilpotent Lie groups the heat semigroup is a semigroup of kernel operators on L^p , $1 \leq p < \infty$ and on C_0 . Cygan [1] has obtained formulas for heat kernels for any two step nilpotent simply connected Lie group. Cygan found the heat kernel for a free simply connected two step nilpotent group, G , using the representation of $L^1(G)$ obtained from the irreducible unitary representation of G in a Hilbert space. He then obtained the heat kernel for a general two step nilpotent Lie group by the "method of descent".

We introduce a more direct method for finding the heat kernel for an H -type group using the fact that the heat kernel for the Heisenberg group can be regarded as the Radon transform of the heat kernel for an H -type group together with the heat kernel for the Heisenberg group obtained by Hulanicki [4] and Gaveau [3]. This method was introduced by Ricci [8].

Jørgensen [5] and Kiszyński [7] have shown that the heat semigroup can be extended to a holomorphic family of operators

$$\{H_z : z \in \mathbb{C}, \operatorname{Re}(z) > 0\} \quad \text{on} \quad L^p(G), \quad 1 \leq p \leq \infty,$$