# INEQUALITIES FOR THE JOINT SPECTRUM OF SIMULTANEOUSLY TRIANGULARIZABLE MATRICES

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## 1. INTRODUCTION

Let $A = (A_1, ..., A_m)$ be an m-tuple of n by n matrices. We say that A is *triangularizable* if there is an invertible matrix Q such that $Q^{-1}A_jQ$ is (upper) triangular for each $j = 1, ..., m$. In this case, for $1 \le k \le n$, let $\alpha_j^{(k)} = (Q^{-1}A_jQ)_{kk}$ the (k, k) element of $Q^{-1}A_jQ$, and set $\alpha^{(k)} = (\alpha_1^{(k)}, ..., \alpha_m^{(k)}) \in \mathbb{C}^m$. The set

$$(1.1) \qquad \sigma(A) = \{\alpha^{(k)} : 1 \le k \le n\}$$

is called the *joint spectrum* of A. For a discussion of this spectrum see Pryde [16].

In particular $\sigma(A)$ has an important subset $\sigma_{pt}(A)$, the *joint point spectrum*, whose elements $\lambda = (\lambda_1, ..., \lambda_m)$ satisfy $A_jx = \lambda_jx$ for all j and some non-zero $x \in \mathbb{C}^n$. We say that $\lambda$ is a *joint eigenvalue* of A with corresponding *joint eigenvector* x. If the $A_j$ commute then $\sigma(A) = \sigma_{pt}(A)$, though this is not the case in general. However, by a theorem of Lie, if A is triangularizable then $\sigma_{pt}(A)$ is non-empty.

Our aim in this paper is to investigate perturbation inequalities for the joint spectra of triangularizable m-tuples. For this purpose we define the function S(K, L) on compact subsets K and L of $\mathbb{C}^m$ by

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