A CONTINUITY PROPERTY RELATED TO AN INDEX OF NON–WCG AND ITS IMPLICATIONS

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Consider a set–valued mapping Φ from a topological space A into subsets of a topological space X. Then Φ is said to be *upper semi–continuous* at $t \in A$ if given an open set W in X containing $\Phi(t)$ there exists an open neighbourhood U of t such that $\Phi(U) \subseteq W$. For brevity we call Φ an *usco* if it is upper semi–continuous on A and $\Phi(t)$ is a non–empty compact subset of X for each $t \in A$. If X is a linear topological space we call Φ a *cusco* if it is upper semi–continuous on A and $\Phi(t)$ is a non–empty convex compact subset of X for each $t \in A$. An usco (cusco) Φ from a topological space A into subsets of a topological (linear topological) space X is said to be *minimal* if its graph does not strictly contain the graph of any other usco (cusco) with the same domain.

For a bounded set E in a metric space X, the *Kuratowski index of non–compactness* is

$$\alpha(E) \equiv \inf\{r > 0 : E \text{ is covered by a finite family of sets of diameter less than } r\}.$$

It is well known that if X is complete then $\alpha(E) = 0$ if and only if E is relatively compact, [6, p.303].

In a recent paper by Giles and Moors [4], a new continuity property related to Kuratowski's index of non–compactness was examined. In that paper they said that a set–valued mapping Φ from a topological space A into subsets of a metric space X is *α upper semi–continuous* at $t \in A$ if given $\varepsilon > 0$ there exists an open neighbourhood U of t such that $\alpha(\Phi(U)) < \varepsilon$. They showed that if the subdifferential mapping of a continuous convex function ϕ on an open convex subset of a Banach space is α upper semi–continuous on a dense subset of its domain then ϕ is Fréchet differentiable on a dense and G_δ subset of its domain. This result led to the consideration of two generalisations of Kuratowski's index of non–compactness.

For a set E in a metric space X the *index of non–separability* is

$$\beta(E) \equiv \inf\{r > 0 : E \text{ is covered by a countable family of balls of radius less than } r\},$$

when E can be covered by a countable family of balls of a fixed radius, otherwise, $\beta(E) = \infty$. Further $\beta(E)= 0$ if and only if E is a separable subset of X, [7].

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