REPRESENTATIONS OF COMPACT GROUPS, CUNTZ-KRIEGER ALGEBRAS, AND GROUPOID C^*-ALGEBRAS(*)

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Doplicher and Roberts have recently showed how to recover a compact Lie group G from a single faithful representation ρ of G in $SU_n(\mathbf{C})$, via a C^*-algebra $\mathcal{O}_\rho$, constructed from the intertwiners of the tensor powers of ρ, and an endomorphism of $\mathcal{O}_\rho$ [3, 4]. The key idea is that the tensor powers ρ^n contain every irreducible representation $\pi \in \hat{G}$, so their intertwiners should contain information about the decompositions of $\pi_1 \otimes \pi_2$ for all $\pi_i \in \hat{G}$, and hence characterise G. We found it intriguing that the theory is based on just one representation ρ, apparently randomly chosen, and attempted to understand how this works. As a first step, we investigated the structure of the algebra $\mathcal{O}_\rho$, and how it depends on ρ.

Our first plan was to identify $\mathcal{O}_\rho$ as the C^*-algebra of a locally compact groupoid $\mathcal{P}$, and exploit the theory of groupoid C^*-algebras [7]. Since $\mathcal{O}_\rho$ is constructed from finite-dimensional pieces, and in particular has a large AF core, we looked at the Bratteli diagram of this core. It has a good deal of vertical symmetry — indeed, one can identify the vertices at each level with the set $\hat{G}$. Thus the path space X of the diagram carries a natural shift, and the groupoid $\mathcal{P}$ is a subset of the groupoid semidirect product $X \times X \times \mathbf{Z}$ with an appropriate topology. Next, we noticed that by enlarging the path space X, we obtained a similar groupoid whose C^*-algebra was the Cuntz-Krieger

(*) This research was supported by the Australian Research Council.

Amer. Math. Soc. 1991 Mathematics Subject Classification: 22D25, 46L80.