DIFFERENTIABILITY PROPERTIES OF BANACH SPACES WHERE THE BOUNDARY OF THE CLOSED UNIT BALL HAS DENTING POINT PROPERTIES

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It was Collier, [2] who showed that for a Banach space with the Radon–Nikodym Property, a continuous convex function on an open convex domain in the dual space is Fréchet differentiable on a dense G_δ subset of its domain provided that the set of points where the function has a weak * continuous subgradient is dense in its domain. The separable Banach space c_0 does not have the Radon–Nikodym Property and the norm of its dual ℓ_1 is nowhere Fréchet differentiable, [11, p.80]. Nevertheless, it has recently been shown that a large class of Banach spaces which includes the weakly compactly generated spaces do have comparable differentiability properties to those of Banach spaces with the Radon–Nikodym Property. Kenderov and Giles, [7, Theorem 3.5], showed that for a Banach space which can be equivalently renormed so that every point on the boundary of the closed unit ball is a denting point, a continuous convex function on an open convex domain in the dual space, is Fréchet differentiable on a dense G_δ subset of its domain provided that the set of points where the function has a weak * continuous subgradient is residual in its domain.

This result was extended by the authors using a generalisation of the notion of denting point, firstly by Kuratowski's index of non–compactness, [5, Theorem 4.5] and secondly by de Blasi's weak index of non–compactness, [6, Theorem 4.3]. Generalising the notion of denting point by an index of non–separability, Moors made a further extension, [9, Theorem 5.6].

In this paper we introduce yet another generalisation of the notion of denting point by an index more general than those so far introduced. We present the Kenderov and Giles theorem in the most general form given so far, and one which includes all the earlier extensions.

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