

CLIFFORD MARTINGALES,
THE $T(b)$ THEOREM
AND CAUCHY INTEGRALS

Garth I. Gaudry

1. INTRODUCTION

Clifford analysis has proved in the last few years to be particularly useful in the study of singular integrals on curves and surfaces [13], [14], [16], [17], [18]. While Clifford algebras have enjoyed a deal of popularity and success in mathematical physics [2, Notes to Chap. 1], [11] and were a subject of much attention from the 1930's onwards [20], the impetus for using them in the study of analysis on curves and surfaces is recent, and due to R. Coifman.

The purpose of this article is to give some indications of recent work by the author, in collaboration with R. Long and T. Qian, on Clifford martingales and their application to the proof of a suitable version of the $T(b)$ theorem for Clifford-valued functions, and to the L^2 -boundedness of the Cauchy principal value integral on Lipschitz surfaces. The full details are to appear elsewhere. The results are not new, but the methods of proof are. They are inspired by the paper of Coifman, Jones and Semmes [3]. One half of their paper gives a proof of the boundedness of the Cauchy integral on Lipschitz curves using dyadic partitions with respect to arc-length, and the corresponding Haar functions and martingales. There are significant differences in the present context, in that it is necessary to construct a novel dual system of left- and right-martingales in order to cope with the noncommutativity of the Clifford algebra. Full details are to appear elsewhere [11].

It is a pleasure to acknowledge the influence of Michael Cowling and Alan McIntosh, who encouraged us to carry through this programme of research.