THE FRÉCHET DIFFERENTIABILITY OF CONVEX FUNCTIONS ON $C(S)$

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INTRODUCTION

In this paper we continue our study of the differentiability of convex functions with domain in a topological linear space: here we are particularly concerned with Fréchet differentiability and the space $C(S)$, where S is an arbitrary topological space. The maximum functions, defined by $m_A = \sup_{t \in A} x(t)$ ($A \subset S$ compact), are good test functions: we show that the Fréchet differentiability points of m_A are precisely those functions which attain their maximum on A at a single isolated point of A; m_A is Fréchet differentiable on a dense subset of $C(S)$ if and only if the set of isolated points of A is dense in A. Relationships between the seminorms p_A and the corresponding maximum functions are given. The set of Fréchet differentiability points of m_A, and hence of p_A, is open; this generalises the well known result that for compact S the sup norm on $C(S)$ is Fréchet differentiable on an open set. In a topological section, connections between the structure of thin and dispersed sets lead into an exploration of the topology of S for Asplund $C(S)$ spaces. In particular, we prove that $C(\mathbb{Q})$ is an Asplund space.

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