ON KAKUTANI'S CRITERION AND SHIRYAEV'S THEOREM

A.H. Dooley

Kakutani's classical "dichotomy" result gives a criterion for when two product measures $\mu = \bigotimes_{i=1}^{\infty} \alpha_i$ and $\nu = \bigotimes_{i=1}^{\infty} \beta_i$ on the infinite product space $\prod_{i=1}^{\infty} \mathbb{Z}_2$ are absolutely continuous. Here, for each i, α_i and β_i are probability measures on the two-point space $\mathbb{Z}_2$. In fact, it turns out that either $\mu \prec \nu$, in the case where $\sum |\alpha_i(0) - \beta_i(0)|^2 < \infty$, or else $\mu \perp \nu$. (See [4]).

Similar results were obtained by Brown and Moran [3] and Peyrière [5] for Riesz products on the circle $\mathbb{T}$. They showed that if

$$\mu = w^*\text{-}\lim \prod_{i=1}^{k} (1 + a_i \cos(3^i t + \phi_i))dt$$

and

$$\nu = w^*\text{-}\lim \prod_{i=1}^{k} (1 + b_i \cos(3^i t + \psi_i))dt$$

with $a_i, b_i \in (-1, 1)$,

then $\mu \sim \nu \quad iff \quad \sum |a_i e^{i\phi} - b_i e^{i\psi}|^2 < \infty$, and otherwise $\mu \perp \nu$.

A far-reaching generalization of Kakutani's theorem is discussed in [7], where we consider a measurable space $(\Omega, \mathcal{C})$ equipped with a non-decreasing family $(\mathcal{C}_n)_{n \geq 0}$ of σ-algebras such that $\mathcal{C} = \vee_n \mathcal{C}_n$.

Suppose that μ and ν are two probability measures on $(\Omega, \mathcal{C})$ such that their re-

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