# SOME PROPERTIES OF SYMMETRIC OPERATOR SPACES

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**0. Introduction** In this paper, we discuss certain aspects of the theory of rearrangement invariant Banach spaces of measurable operators affiliated with a semi-finite von Neumann algebra, which have been the subject of recent work by the present authors and Ben de Pagter. Such a theory provides a unified approach to the study of trace ideals initiated by Schatten [Sch] and to the study of rearrangement invariant (commutative) Banach function spaces which play a central role in classical real analysis, and derives its motivation from each of these central sources. The first general construction of such spaces, based on real analysis methods and using the theory of non-commutative integration developed by Segal [Se] (see also Dixmier [Dix]), is due to Ovčinnikov [Ov 1,2] and, independently, to Yeadon [Ye1]. More recently, an approach to the construction of symmetric operator spaces has been given in [DDP1,2] at a level of generality that fully reflects the commutative theory. The relation of this construction to many theorems of classical interpolation theory has been given in [DDP3]. In the present survey, our attention will be directed primarily towards the development of a general duality theory and related topological and geometrical properties.

After gathering the necessary preliminaries in Section 1, we outline in Section 2 the principal results concerning Köthe duality obtained in [DDP4]. The results presented in this section find their principal motivation in the well-known theory of Banach function spaces, and considerably extend and refine earlier investigations of Garling [Ga1,2] and Yeadon [Ye2]. The central theme of Section 3 is the study of weakly compact subsets of symmetric operator spaces and its relation to the characterization of several structural and topological properties, such as weak sequential completeness and reflexivity. Full details and proofs of the results of this section will appear in [DDP4,5]. Section 4 combines results

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