SOME NEW APPLICATIONS OF RIESZ PRODUCTS

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1. INTRODUCTION.

The results which I will describe arise from joint work with Bill Moran, and with Dani Berend, Charles Pearce and Andy Pollington. The common theme is the formalism of Riesz product measures, for these have proved to be exceptionally useful in the study of normal numbers. Elsewhere in these proceedings, Tony Dooley describes some of our joint work on the related class of G-measures (G for Gibbs) which we introduced, following work of Keane, [6], to generalize certain important features of Riesz products. I am confident that G-measures will also have significant applications – but, curiously, it is those properties of Riesz products which are sacrificed in that generalization which are the ones which matter here.

In fact the simplest case is when m denotes Haar measure on the circle $\mathbb{T}$ and we consider

$$\mu = \lim_{n\to\infty} \prod_{n=1}^{N} (1 + \cos 2\pi t^n x).m(dx),$$

for some integer t greater than 3. The limit is taken in the weak * topology (evaluation on continuous functions) and the resulting measure μ is a probability distribution i.e. is positive and has mass one. The important fact is that the Fourier transform $\hat{\mu}$ vanishes off words of the form

$$\sum_{i=1}^{M} \epsilon_i t^i, \quad \epsilon_i \in \{0, \pm 1\}$$

1991 *Mathematics Subject Classification.* 42A55.