SPECTRUM OF THE RUELLE OPERATOR AND ZETA FUNCTIONS FOR BROKEN GEODESIC FLOWS

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1. Introduction

Let K be an obstacle in $\mathbb{R}^n$, where $n \geq 3$ is odd, i.e. K is a compact subset of $\mathbb{R}^n$ with C^∞ boundary ∂K such that

$$\Omega = \overline{\mathbb{R}^n \setminus K}$$

is connected. One of the main objects of study in scattering theory (by an obstacle) is the so called *scattering matrix* $S(z)$ related to the wave equation in $\mathbb{R} \times \Omega$ with Dirichlet boundary condition on $\mathbb{R} \times \Omega$. This is (cf. [LP], [M] or [Z]) a meromorphic operator-valued function

$$S(z) : L^2(\mathbf{S}^{n-1}) \longrightarrow L^2(\mathbf{S}^{n-1})$$

with poles $\{\lambda_j\}_{j=1}^{\infty}$ in the half-plane $\mathrm{Im}(z) > 0$.

A variety of problems in scattering theory deal with finding geometric information about K from the distribution of the poles $\{\lambda_j\}$. In what follows we describe one particular problem of this type.

The obstacle K is called **trapping** if there exists an infinitely long bounded broken geodesic (in the sense of Melrose and Sjöstrand [MS]) in the *exterior domain* Ω. For example, if Ω contains a periodic broken geodesic (this is always the case when K has more than one connected component), then K is trapping.

It follows from results of Lax-Phillips (1971) and Melrose (1982) that if K is non-trapping, then $\{z : 0 < \mathrm{Im}(z) < \alpha\}$ contains finitely many poles λ_j for any $\alpha > 0$ (cf. the Epilogue in [LP] for more precise information).

In the first edition of their monograph *Scattering Theory* published in 1967, Lax and Phillips conjectured that for trapping obstacles there should exist a sequence $\{\lambda_j\}$ of scattering poles such that $\mathrm{Im}\lambda_j \to 0$ as $j \to \infty$. However M. Ikawa [I1] showed that this is not the case when K is a disjoint union of two strictly convex compact domains with smooth boundaries. It turned out that in this particular case the scattering matrix has poles approximately at the points $\dfrac{k\pi}{d} + i\delta$, $k = 0, \pm 1, \pm 2, \ldots$, where d is the distance between the two connected