3. DIRECT PRODUCTS WITH FINITE FACTORS

This section contains the fundamental results of the present work. They concern primarily direct decompositions of finite algebras; as we shall see, however, most of them also apply to direct decompositions of arbitrary algebras -- under the assumption that some of the factors involved are finite.

The notion of an *indecomposable algebra* will play an important part in our discussion. We define:

Definition 3.1. *An algebra*

$$\underline{A} = \langle A, +, O_0, O_1, \ldots, O_\xi, \ldots \rangle$$

is said to be indecomposable if $A \neq \{0\}$ *and for any subalgebras* B *and* C *of* A, $A = B \times C$ *implies that* $B = \{0\}$ *or* $C = \{0\}$.

Corollary 3.2. *For every finite algebra*

$$\underline{A} = \langle A, +, O_0, O_1, \ldots, O_\xi, \ldots \rangle$$

there exist indecomposable subalgebras $A_0, A_1, \ldots, A_\kappa, \ldots$ *with* $\kappa < \nu < \omega$ *such that*

$$A = \prod_{\kappa<\nu} A_\kappa.$$

Proof: obvious (by induction or by contradiction).

We now give two auxiliary theorems which concern homomorphisms of finite central subalgebras of arbitrary algebras.[13] In formulating and proving these theorems we shall use the familiar notions of the $\varkappa$-th *iteration* f^κ of a function f. This notion is understood to be defined recursively in terms of that of the composition fg of two functions f and g; by f^0 we understand the identity function (possibly with the domain restricted to that of f), and we put

$$f^{\kappa+1} = f^\kappa f \text{ for every } \varkappa < \omega.$$

13. These theorems have been established for groups by H. Fitting; cf. Fitting [1], pp. 19 f.