1. ELEMENTARY PROPERTIES OF DIRECT PRODUCTS

We shall concern ourselves with systems

$$\underline{A} = \langle A, +, O_0, O_1, \ldots, O_\xi, \ldots \rangle$$

constituted by an arbitrary set A, a binary operation + (the operation of addition), and arbitrarily many other operations arranged in a sequence $O_0, O_1, \ldots, O_\xi, \ldots$ of a type τ (where τ is a finite or transfinite ordinal). Each of these operations O_ξ is assumed to be defined for finite or transfinite sequences of elements $x_0, x_1, \ldots, x_\kappa, \ldots$ of a well-determined type ρ_ξ called the rank of the operation. Thus, O_ξ may be a unary operation ($\rho_\xi = 1$), a binary operation ($\rho_\xi = 2$), a ternary operation ($\rho_\xi = 3$), an operation on simple infinite sequences ($\rho_\xi = \omega$), etc. An operation O_ξ with the rank $\rho_\xi = \mu$ will be referred to for brevity as a μ-ary operation. Two systems

$$\underline{A} = \langle A, +, O_0, O_1, \ldots, O_\xi, \ldots \rangle \text{ with } \xi < \tau$$

and

$$\underline{A}' = \langle \ldots, +', O_0', O_1', \ldots, O_\xi', \ldots \rangle \text{ with } \xi < \tau'$$

are called similar if the sequences of ranks ρ_ξ and ρ_ξ' are identical, i.e., if

$$\tau = \tau', \text{ and } \rho_\xi = \rho_\xi' \text{ for every } \xi < \tau$$

The sequence of ranks ρ_ξ will sometimes be referred to as the similarity type of the system $\underline{A}$.

The symbolic expression

$$x, y, \ldots \varepsilon A$$

will express, as usual, the fact that x, y,... are elements of A. We shall speak occasionally of elements of the system $\underline{A}$ having actually in mind elements of the set A; and we shall call the system $\underline{A}$ finite (or infinite) in case the set A is finite (or infinite).