$\mathbb{C}^n$-CAPACITY AND MULTIDIMENSIONAL MOMENT PROBLEM

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Introduction

Let K be a compact set in the n-dimensional complex space $\mathbb{C}^n$, $H(K)$ be a space of holomorphic functions on K, $H'(K)$ be the space of linear continuous functionals over $H(K)$. We will write down the value of the functional $\mu \in H'(K)$ on the function $h \in H(K)$ in the form of $<\mu, h>$. The numbers of the form $C_\nu(\mu) = <\mu, Z^\nu>$ are called the moments of the analytical functional μ, where $Z^\nu = Z_1^{\nu_1} \dots Z_n^{\nu_n}$ is a holomorphic monomial of the degree $|\nu| = \nu_1 + \dots + \nu_n$; $Z = (Z_1, \dots, Z_n) \in \mathbb{C}^n$, $\nu = (\nu_1, \dots, \nu_n) \in \mathbb{Z}_+^n$.

The problem arising from a number of applications (computational tomography [1], inverse problem of the potential theory [2], quadrature formulae [3], and even production functions theory [4]) is to reconstruct a functional from $H'(K)$ through its moments.

The necessary and sufficient condition of uniqueness of a functional $\mu \in H'(K)$, which has the fixed moments $\{C_\nu(\mu)\}$ is polynomial convexity of the compact set K, since polynomial convexity of K is necessary and sufficient in order that any function from $H(K)$ will be approximated by holomorphic polynomials (A. Weil, 1932).

If a functional μ is given by positive measure on the compact set $K \subset R^n \subset \mathbb{C}^n$ then the considered problem is called the classical moment problem. This classical problem is effectively and completely solved only for the case $n = 1$ (see [5]).

In connection with applications the problem of the approximate reconstruction of the functional $\mu \in H'(K)$ through the finite number of moments C_ν, $|\nu| \leq N$ is of particular interest. In the classical theory this problem is called the Markov moment problem. In order to solve this problem it is necessary to answer at least the following questions: