ABSTRACT SET THEORY

by

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1. Historical remarks. Outlines of Cantor's theory

Almost 100 years ago the German mathematician Georg Cantor was studying the representation of functions of a real variable by trigonometric series. This problem interested many mathematicians at that time. Trying to extend the uniqueness of representation to functions with infinitely many singular points he was led to the notion of a derived set. This was not only the beginning of his study of point sets but lead him later to the creation of transfinite ordinal numbers. This again lead him to develop his general set theory. The further development of this, the different variations or modifications of it that have been proposed in more recent years, the discussions and criticisms with regard to this subject, will constitute the contents of my lectures on set theory.

One ought to notice that there have been some anticipations of Cantor's theory. For example B. Bolzano wrote a paper with the title: Paradoxien des Unendlichen (1951) (Paradoxes of the Infinite), where he mentioned some of the astonishing properties of infinite sets. Already Galilei had noticed the remarkable fact that a part of an infinite set in a certain sense contained as many elements as the whole set. On the other hand it ought to be remarked that about the same time that Cantor exposed his ideas some other people were busy in developing what we today call mathematical logic. These investigations concerned among other things the fundamental notions and theorems of mathematics, so that they should naturally contain set theory as well as other more elementary or ordinary parts of mathematics. A part of the work of another German mathematician, R. Dedekind, was also devoted to studies of a similar kind. In particular, his book "Was sind und was sollen die Zahlen" belongs hereto.

In my following first talks I will however confine my subject to just an exposition of the most characteristic ideas in Cantor's work, mostly done in the years 1874-97.

The real reason for a mathematician to develop a general set theory was of course the fact that in mathematics we often have to do not only with single mathematical objects but also with collections of them. Therefore the study of properties of such collections, even infinite ones, must be of very great importance.

There is one fact to which I would like to call attention. Most of mathematics and perhaps above all the classical set theory has been developed in accordance with the philosophical attitude called Platonism. This standpoint means that we consider the mathematical objects as existing before and independent of our actual thinking. Perhaps an illustrating way of expressing it is to say that when we are thinking about mathematical objects we are looking at eternal preexisting objects. It seems clear that the word "existence"