INTRODUCTION

The central concept in these lectures is that of a formal system. This is quite a different notion from that of a postulate system, as naively conceived a half century ago. In the older conception a mathematical theory consisted of a set of postulates and their logical consequences. The trouble with that idea is that no one knows exactly what a logical consequence is, and the paradoxes have shown that our intuitive feelings on the subject are not reliable. In the modern conception this vague and subjective notion is replaced by the objective one of derivability according to explicitly stated rules.

A formal system is a body of propositions, which we shall call elementary propositions, concerning which we have a very precise and objective criterion of truth. This criterion has a recursive character. We start with a set of these propositions the axioms - which are stated to be true outright as part of the definition of the system; and to these we add explicit rules for deriving further true propositions from those already established It is then understood that an elementary proposition is a theorem - i.e., is true - if and only if it is an axiom or is derived from the axioms by the rules. It is further required that the specifications as to rules, axioms, etc., be definite, in the sense that there be a finite constructive process for deciding in any given case whether the concept applies or not. Thus the truth of an elementary proposition, although not necessarily itself a definite concept (the system has a relatively trivial character when it is), is nevertheless precise and objective in that the checking of evidence for it - i.e., of a proof - is a definite process.

This notion of formal system is fundamental, in one form or another, to many types of modern logical investigations. Frequently the systems studied have to do with symbols as subject matter. Thus in Hilbert's metamathematics the elementary propositions are of the form

"a is provable"

where a is a certain type of combination of the symbols of an exactly specified "language." But that is not essential. One can, if one likes, set up high-school algebra as a formal system, in which the elementary propositions are the equations

$$a = b,$$