2. THE CENTER OF AN ALGEBRA

The study of groups has clearly shown that the properties of their direct decompositions depend to a large extent on those of their center or, in the case of groups with a set Ω of operators, on those of what is called the Ω-center.[9] This applies also to arbitrary algebras in the sense of 1.1; however, the definition of a center is in this case more involved. The center of an algebra will be defined (in 2.10) as the set-theoretical union of certain subalgebras which are referred to as central subalgebras.

Definition 2.1. A subalgebra C of an algebra

$$\underline{A} = \langle A, +, O_0, O_1, \ldots, O_\xi, \ldots \rangle$$

is called a central subalgebra if it satisfies the following conditions:

(i) If $c \in C$, then there exists an element $\bar{c} \in C$ such that

$$c + \bar{c} = 0;$$

(ii) If $a_1, a_2 \in A$ and $c_1, c_2 \in C$, then

$$(a_1 + c_1) + (a_2 + c_2) = (a_1 + a_2) + (c_1 + c_2);$$

(iii) If O_ξ is a μ-ary operation, and if $a_0, a_1, \ldots, a_\kappa, \ldots \in A$ and $c_0, c_1, \ldots, c_\kappa, \ldots \in C$ for $\kappa < \mu$, then

$$O_\xi(a_0 + c_0, a_1 + c_1, \ldots, a_\kappa + c_\kappa, \ldots) = O_\xi(a_0, a_1, \ldots, a_\kappa, \ldots) + O_\xi(c_0, c_1, \ldots, c_\kappa, \ldots).$$

Conditions (ii) and (iii) of this definition are closely related to conditions (iii) and (iv) of Definition 1.4; this circumstance will play an essential part in further developments. 2.1 (ii) can clearly be replaced by condition (ii) of Theorem 2.2 below. In case the rank μ of an operation O_ξ is finite, 2.1 (iii) is easily seen to be equivalent to each of the following conditions:

(iii') If $a_0, a_1, \ldots, a_\kappa, \ldots \in A$ and $c_0, c_1, \ldots, c_\kappa, \ldots \in C$ for $\kappa < \mu$, then

$$O_\xi(a_0 + c_0, a_1 + c_1, \ldots, a_\kappa + c_\kappa, \ldots) = O_\xi(a_0, a_1, \ldots, a_\kappa, \ldots) + \sum_{\kappa<\mu} O_\xi(0, 0, \ldots, 0, c_\kappa, 0, \ldots).$$

9. See, e.g., Speiser [1], p. 30, for groups without operators, and Kořínek [1], p. 273, for groups with operators.