Definition 5.1. Let k be a field, A an overring of k. The ring A is said to be geometrically regular if, for all finite field extensions k' of k, the ring $A' = A \otimes_k k'$ is regular.

Corollary 5.3. a) Every regular overring of a perfect field is geometrically regular. .

b) Every regular overring of an algebraically closed field is geometrically regular.

Remark. Let again $A' = A \otimes_k k'$. Some of the properties of A' can be deduced from those of A and of the field extension k' of k. This process of deduction is known as ascent. Conversely, some of the properties of A can be deduced from those of A'. This latter process of deduction is known as descent.

§6. COMPLETION AND NORMALIZATION

6A. Completion. Let A be a noetherian local ring, $\mathfrak{m}$ its maximal ideal. It is well known(see Corollary after Proposition 5 in B.C.A., III, §3, no. 2) that $\bigcap \mathfrak{m}^n = (0)$. This implies that the collection $\{\mathfrak{m}^n\}$ can be taken as the basis of a filter of neighborhoods of 0 in a (unique) Hausdorff topology which is consistent with the ring structure of A (i.e. A is a Hausdorff topological ring).

The set $\hat{A}$ of (equivalence classes of) Cauchy sequences of elements of A can be given a topological ring structure which is obviously complete (i.e. every Cauchy sequence in $\hat{A}$ is convergent). We refer the reader to the third chapter of B.C.A. for the proof of the above statements, as well as for the