Some Recent Results and Problems in the Theory of Value-Distribution

Lo Yang

Dedicated to Professor Wilhelm Stoll on the occasion of his inauguration as the Duncan Professor of Mathematics.

For meromorphic functions of one complex variable, the theory of value-distribution has tremendously developed already since the twenties of this century. Although it has a long history, there are still some interesting and remarkable results during the recent years. For instance, Drasin [6] proved that the F. Nevanlinna conjecture is correct; Lewis and Wu [13] made a significant step in proving the Arakelyan's conjecture [1]; Osgood [18] and Steinmetz [20] independently proved the defect relation for small functions, and so on.

In this lecture, I would like to mention some recent results and problems which are based on my own interests.

1. Precise estimate of total deficiency of meromorphic functions and their derivatives

Let $f(z)$ be a transcendental meromorphic function in the finite plane and a be a complex value (finite or infinite). According to R. Nevanlinna

$$\begin{aligned}\delta(a,f) &= \liminf_{r\to\infty} \frac{m\left(r,\frac{1}{f-a}\right)}{T(r,f)} \\ &= 1-\limsup_{r\to\infty}\frac{N\left(r,\frac{1}{f-a}\right)}{T(r,f)}\end{aligned}$$

It is clear that $0 \le \delta(a,f) \le 1$. If $\delta(a,f)$ is positive, then a is named a deficient value with respect to $f(z)$ and $\delta(a,f)$ is its deficiency. The most fundamental result of Nevanlinna theory can be stated as follows [12, 17, 23].