DIOPHANTINE APPROXIMATION AND THE THEORY OF HOLOMORPHIC CURVES

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In the last few years, due to the works of Osgood [**O1,2**], Lang [**L1,2,3**], Vojta [**V1,2,3,4**] and others, there appear to be evidences that the Theory of Diophantine Approximation and The Theory of Holomorphic curves (Nevanlinna Theory) may be somehow related. Currently, the relationship between the two theories is still on a formal level even though the resemblance of many of the corresponding results is quite striking. Vojta has come up with a dictionary for translating results from one theory into the other. Again the dictionary is essentially formal in nature and seems somewhat artificial at this point, it is perhaps worthwhile to begin a systematic investigation. Recently, I began to study the Theory of Diophantine Approximations, with the motivation of formulating the theory so that it parallels the theory of curves. These notes is a (very) partial survey of some of the results in diophantine approximations and the corresponding results in Nevanlinna Theory.

(I) Diophantine Approximation

The theory of diophantine equations is the study of solutions of polynomials over number fields. Typically, results in diophantine equations come in the form of certain finiteness statements; for instance statements asserting that certain equations have only a finite number of rational or integral solutions. We begin with a simple example.

Example 1 Consider the algebraic variety $X^2 + Y^2 = 3Z^2$ in $\boldsymbol{P}^2$, we claim that there is no rational (integral) points (points with rational (integral) coordinates; on projective spaces a rational point is also an integral point) on this variety. To see this, suppose $P = [x, y, z]$ be a rational point on the variety with $x, y, z \in \boldsymbol{Z}$ and $gcd(x, y, z) = 1$.

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