RECENT WORK ON NEVANLINNA THEORY AND DIOPHANTINE APPROXIMATIONS

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What I will describe here is a formal analogy between value distribution theory and various diophantine questions in number theory. In particular, there is a dictionary which can be used to translate, e. g., the First and Second Main Theorems of Nevanlinna theory into the number field case. For example, we shall see that the number theoretic counterpart to the Second Main Theorem combines Roth's theorem and Mordell's conjecture (proved by Faltings in 1983).

This analogy is only formal, though: it can only be used to translate the statements of main results, and the proofs of some of their corollaries. The proofs of the main results, though, cannot be translated due to a lack of a number theoretic analogue of the derivative of a meromorphic function, among other reasons. All that I can say at this point is that negative curvature plays a role in the proofs in both cases.

Thus, until recently the analogy was good only for producing conjectures, by translating statements of theorems in value distribution theory into number theory. But in 1989 it has played a role in finding a new proof of the Mordell conjecture, via the suggestion that the Mordell conjecture and Roth's theorem should have a common proof, as is the case with the Second Main Theorem.

We begin by briefly describing this analogy, but only briefly as it has been described elsewhere in [**V 1**] and [**V 2**], as well as in the book [**V 3**]. Likewise, more recent results will be described in [**V 6**]; therefore we refer the reader to [**V 3**] and [**V 6**] for details.

Let $f : \mathbb{C} \to C$ be a holomorphic curve in a compact Riemann surface (which we may assume is connected). Let D be an effective reduced divisor on C; i.e., a finite set of points, and let $\operatorname{dist}(D, P)$ be some function measuring the distance from P to a fixed divisor D. Then we have the usual definition

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