NEVANLINNA THEOREMS IN PUSH-FORWARD VERSION

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I. Introduction

We consider a polynomial map f of $\mathbb{C}^n$, i.e., a holomporphic map $f : \mathbb{C}^n_z \to \mathbb{C}^n_w, z = (z_1, \ldots, z_n) \mapsto (f_1(z), \ldots, f_n(z))$, where $\mathbb{C}^n_z = \mathbb{C}^n_w = \mathbb{C}^n, z = (z_1, \ldots, z_n)$ and $w = (w_1, \ldots, w_n)$ are the coordinate systems for $\mathbb{C}^n_z$ and $\mathbb{C}^n_w$, respectively, and $f_1, \ldots, f_n \in \mathbb{C}[z_1, \ldots, z_n]$. For any polynomial map $f : \mathbb{C}^n_z \to \mathbb{C}^n_w$ with $\det(Df) \neq 0$, it is naturally associated a dominant rational map $F : \mathbb{P}^n_z - - \to \mathbb{P}^n_w$ defined by $[z_0 : z_1 : \ldots : z_n] \mapsto [z_0^{\deg f} : F_1(z_0, z_1, \ldots, z_n) : \ldots : F_n(z_0, \ldots, z_n)]$, where F_i is the homogeneous polynomial of degree $\deg f := \max_{1 \leq t \leq n} \deg f_t$ uniquely determined by $F_i(1, z_1, \ldots, z_n) = f_i(z_1, \ldots, z_n)$ for $i = 1, 2, \ldots, n$.

There is the well-known Jacobian problem which was raised by Keller in 1939 [**K**] and is still unknown (cf.[**V**]): If $f : \mathbb{C}^n_z \to \mathbb{C}^n_w$ is a polynomial map with the Jacobian $\det(Df) = 1$, then f has an inverse of polynomial map. In [**J**, corollary 4], we have proved: Let $f : \mathbb{C}^n_z \to \mathbb{C}^n_w$ be a polynomial map with $\det(Df) = 1$. Then f has an inverse of polynomial map if and only if $\text{supp} F_* D_{z_0} = \text{supp} D_{w_0}$, where D_{z_0} is the divisor given by $z_0 = 0$ and $F_* D_{z_0}$ is the push-forward current which is indeed a divisor.

From the above result, it leads us to take attention to the push-forward divisor $F_* D_{z_0}$. In order to investigate general push-forward divisors, in this paper, we establish the Nevanlinna main theorems in push-forward version which are analogous to the ones in the value distribution theory. We shall study any polynomial map $f : \mathbb{C}^n_z \to \mathbb{C}^n_w$ with $\det Df \neq 0$, its associated a dominant rational map $F : \mathbb{P}^n_z - - \to \mathbb{P}^n_w$ and any divisor D on $\mathbb{P}^n_z$. We shall prove the first main theorem, the second main theorem, the defect relation and some other results. For proving these theorems, besides the modified traditional method in the value distribution theory, some estimate from [**J**, theorem 2] about push-forward currents will be used.