THE NEVANLINNA ERROR TERM FOR COVERINGS GENERICALLY SURJECTIVE CASE

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Nevanlinna theory [**Ne**] started as the theory of the value distribution of meromorphic functions. The so-called Second Main Theorem is a theorem relating how often a function is equal to a given value compared with how often, on average, it is close to that value. This theorem takes the form of an inequality relating the counting function and the mean proximity function by means of an error term. Historically, only the order of the error term was considered important, but motivated by Vojta's [**Vo**] dictionary between Nevanlinna theory and Diophantine approximations, Lang and others, see [**La**] and [**L-C**] for instance, have started to look more closely at the form of this error term.

Vojta has a number theoretic conjecture, analogous to the Second Main Theorem, where the absolute height of an algebraic point is bounded by an error term, which is independent of the degree of the point. This caused Lang to raise the question, "how does the degree of an analytic covering of **C** come into the error term in Nevanlinna theory?" The second part of [**L-C**] looks at Nevanlinna theory on coverings in order to answer this question. Noguchi [**No1**], [**No2**], and [**No3**] and Stoll [**St**] are among those who have previously looked at the Nevanlinna theory of coverings.

As part of Vojta's dictionary, the Nevanlinna characteristic function corresponds to the height of a rational point in projective space. For a number field F, there are two notions of height. There is a relative height and an absolute height. Given a point $P = (x_0, \ldots, x_n)$ in $\mathbf{P}^n(F)$, the *relative* height, $h_F(P)$ is defined by

$$h_F(P) = \sum_{v \in S} [F_v : \mathbf{Q}_v] \log \max_j |x_j|_v,$$

where S is the set of absolute values on F, and $[F_v : \mathbf{Q}_v]$ is the local degree. The *absolute* height $h(P)$ is the relative height divided