ON THE EXISTENCE OF ∅-DEFINABLE NORMAL SUBGROUPS OF A STABLE GROUP

Anand Pillay*

There is a family of results concerning the existence of (∅)-definable normal subgroups of a stable group. Namely:

(1) (Berline-Lascar [B-L]). If G is superstable, and

$$U(G) = \omega^{\alpha_1}n_1 + \ldots + \omega^{\alpha_k}n_k + \beta \ (\beta < \omega^{\alpha_k})$$

then G has a normal subgroup K with $U(K) = \omega^{\alpha_1}n_1 + \ldots + \omega^{\alpha_k}n_k$.

(2) (Hrushovski [H]). If G is stable and its generic type is nonorthogonal to a regular type p, then there is a definable normal subgroup K of G such that G/K is "p-internal" and infinite.

(3) (Pillay-Hrushovski [PH]). If G is 1-based and connected then every type $q = stp(a/A)$ $(a \in G)$ is the generic type of a coset of a normal acl(∅)-definable subgroup K of G.

In this expository paper we will prove these results and some variants.

We work throughout over acl (∅) (i.e. we assume acl (∅) = dcl (∅)). G is assumed throughout to be a saturated, stable, connected group. We prove:

Theorem A. Let g be a generic of G (over ∅), and let $X \subset G$ be invariant and internally closed. Then there is a ∅-definable normal $K \subset G$ such that (i) $G/K \subset X$ and (ii) some stationarisation of $tp(g/X)$ is the generic type of a generic coset of K° (the connected component of K).

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